Information Provision in Procurement Auctions with Endogenous Investments *

Haomin Yan†

February 6, 2019

Abstract

When an auctioneer has private preference over bidders' non-price characteristics, whether to disclose that preference to bidders prior to the auction is a non-trivial problem. This paper analyzes an auctioneer's optimal information provision strategy in a procurement auction in which bidders invest in cost-reducing investments before entering the auction. In this paper, I characterize the equilibrium investment strategies of bidders under three different information provision schemes: public disclosure, private disclosure, and concealment of preferences over bidders. I find that pre-auction investments are strategic substitutes among bidders, and providing more information about the auctioneer's preference encourages those favored bidders to invest more, which results in a more dispersed distribution of costs among bidders in the auction. Then I compare the expected revenues in a second-score auction under these three information provision schemes and show that concealment is the optimal strategy with two bidders.

JEL Classification: D44, D82
Keywords: information provision; procurement auctions; investments

*I am very grateful to Professor Lawrence Ausubel, Professor Daniel Vincent and Professor Emel Filiz-Ozbay for continual support and many helpful suggestions through the development of this research.

†Department of Economics, University of Maryland, College Park MD 20742. Email: yan@econ.umd.edu.

1 Introduction

Auctions are used in procurement settings to allocate contracts to suppliers in a variety of markets such as electricity, government securities, and construction rights. In a benchmark model of single-unit procurement auctions, bidders sell identical products with exogenously differentiated production costs. However, many practical procurement markets have two departures from the standard model. First, the suppliers can be horizontally differentiated in their non-price characteristics, and the auctioneer often has preference over non-price characteristics of the product. Second, bidders can often engage in pre-auction cost-reducing investments. This study is motivated by these two distinctive features in many procurement markets.

Existence of product differentiation is common in procurement auctions. Examples of non-price attributes that the auctioneer might care about include product design, input materials, time of completion, reputation of the supplier, etc. (Asker and Cantillon 2008 [1]). Empirical evidence of product differentiation in procurement auctions is also documented in newspapers and previous studies. For example, when two aircraft manufacturing companies Airbus and Boeing competed for a contract from Iberia Airlines, their bids were evaluated together with their product characteristics in the procurement auction. According to the *Wall Street Jounral* on March 10, 2003, Iberia has privately known preferences on several characteristics such as fleet composition of the potential suppliers' products, as it will affect future maintenance cost (Thomas and Wilson 2012 [14]). Under the presence of product differentiation, a supplier's value in the auction not only depends on its production cost but also depends on the auctioneer's privately known preference.

Pre-auction investments among bidders are also common in procurements. For example, prior to bidding for a road construction contract, suppliers can invest in machinery and other equipments to reduce cost. Empirical evidence of pre-auction investments can also be found in previous studies. For example, defense contractors invest substantial resources in R & D before bidding for a government contract (Lichtenberg 1986 [10]; Li et al. 2006 [9]).

Under these two departures from the standard procurement auction model, suppliers face a trade-off between higher sunk investment costs and higher expected return in the auction, and the auctioneer's information provision policy can affect the suppliers' investment strategies. Since each supplier will choose the investment level at which the marginal expected return to investment in the auction equals to the marginal investment cost, and the expected return to investment depends on the auctioneer's valuation over the supplier.

It is well understood how to design an optimal auction mechanism that maximizes the auctioneer's expected revenue given homogenous bidders who enter the auction with private exogenous monetary types (Myerson 1981 [12]). Some studies have explored pre-auction investment incentives with homogenous products (Piccione and Tan 1996 [13]; Bag 1997 [3]; Arozamena and Cantillon 2004 [2]). However, no study has examined suppliers' investment incentives on cost reduction when product differentiation presents. The objective of this study is to investigate the impact of the auctioneer's information provision policy on suppliers' pre-auction investment incentives and the auctioneer's expected revenue when product differentiation presents among suppliers.

In this paper, I assume that the auctioneer can commit to one of the following three information disclosure policies: publicly disclose her private valuations over all suppliers' products; privately disclose her valuation over each supplier's product; or completely conceal her valuations. Then I analyze equilibrium investment strategy of suppliers before entering a second-score sealed-bid procurement auction and compare the expected revenues of auction under these three information provision schemes. The main result of this paper shows that pre-auction investments are strategic substitutes among bidders, and providing more information about the auctioneer's preference encourages those more favored bidders to invest more, which increases cost differentiation among bidders. The main analysis focuses on the case when there are only two bidders and shows that disclosing more information will reduce expected revenue by discouraging the lower quality bidder from investment and giving higher informational rent to the higher quality bidder. I also provide a discussion of the general case when there are more than 2 bidders and show that disclosing more information will increase expected revenue by promoting competition among higher quality bidders when the number of bidders is sufficiently large.

2 Related Literature

This paper is connected to the literature on procurement auctions with differentiated products. Asker and Cantillon (2008) [1] provide a systematic analysis of equilibrium behavior in scoring auctions when suppliers have multi-dimensional types.

Thomas and Wilson (2012) [14] experimentally compare first-price auctions and multilateral negotiations when horizontal product differentiation is introduced into a procurement auction. The major difference between this paper and the previous studies on scoring auctions is that the existing literature on scoring auctions takes product characteristics and cost as different dimensions of each bidder's exogenously given multi-dimensional type, while this paper models product differentiation as assigning each seller a subjective "quality" privately known to the auctioneer and assumes each bidder's cost is endogenously determined by investment.

This paper is also related to the literature studying optimal information release of the auctioneer when the auctioneer owns private information that enters bidders' valuations. Milgrom and Weber (1982) [11] analyze the optimal release of information in an auction with affiliated values and find that it is optimal for the auctioneer to publicly announce her private information. On the other hand, Ganuza (2004) [7] analyzes a horizontally differentiated market in which the auctioneer has private information about product characteristics and bidders have horizontally differentiated preferences over the product space. He shows that when releasing information is costly to the auctioneer, the auctioneer has incentives to release less than efficient level of information. Coleff and Garcia (2014) [4] study the optimal release of information in a procurement auction in which sellers can choose their horizontal product characteristics according to the auctioneer's reported preference. They show that it is not optimal for the auctioneer to send public information to all sellers under presence of entry cost. Closely related to this paper, Colucci et al. (2015) [5] compare the performance of different information provision schemes under first-score auctions and second-score auctions in a model with differentiated bidders whose qualities are private information to the auctioneer. However, they assume bidders' costs are heterogenous and commonly known in the model, while I adopt Dasgupta (1990) [6]'s production model and assume bidders' costs are determined by their own investment decisions and a random variable. In Ganuza (2004) [7]'s model, the auctioneer's information provision will alter the bidders' perception of their own values. In Coleff and Garcia (2014) [4], the auctioneer's information provision will alter the equilibrium profile of bidders' horizontal locations and the number of bidders. In Colucci et al (2015) [5], the auctioneer's information provision will change the bidders' bidding strategies in the first score auction. This paper is different from the above studies in the sense that the auctioneer's information provision will alter the profile of bidder's values by changing their investment incentives.

This paper is also closely related to the strand of literature on studying bidders' pre-auction investment incentives under different auction mechanisms. Most of this literature focus on studying suppliers' investment incentives in sealed-bid auctions for a homogenous product. A common goal of these studies is to compare the equilibrium investment levels induced by the auction mechanism to the socially optimal investment level, and compare the performance of different mechanisms based on their efficiency in inducing pre-auction investments (Piccione and Tan 1996 [13]; Bag 1997 [3]). However, there exists no mechanism that can uniquely implements ex ante efficient investment when suppliers can only make investment decisions simultaneously prior to the auction (Arozamena and Cantillon 2004 [2]; Li et al. 2006 [9]; Hatfield et al. 2015 [8]; Tomoeda 2015 [15]). Different from these previous studies that focus on finding socially-optimal investment-inducing mechanism, the goal of this paper is to find an information provision scheme that maximizes the auctioneer's ex ante expected revenue in a second score auction, given the presence of differentiated sellers and pre-auction investment opportunity.

3 Model

3.1 Environment

An auctioneer wishes to procure one unit of an indivisible product that may come in different varieties. There are N risk-neutral potential suppliers $i \in \{1, 2, \cdots, N\}$ providing imperfect substitutes that feature different varieties of this product[1]. The product characteristic of each supplier is exogenous and observable to the auctioneer. The auctioneer values the specific product of each supplier differently. There are two stages of the game: investment stage and auction stage. The time line of the game is presented as below:

t=1: At the beginning of the investment stage, the auctioneer announces the allocation and payment rules of a second score auction and the information disclosure policy. The auctioneer can choose to publicly announce the entire profile of her valuations to all suppliers, or to privately inform each supplier her value for that supplier, or to conceal this information.

t=2: N suppliers enter the game. The auctioneer observes the product characteristics

[1] In this paper, I use feminine pronoun for the auctioneer and masculine pronouns for bidders.

of each supplier and privately learns her valuation over their products $\{q_i, \}_{i=1}^N$. Each q_i measures the match between the auctioneer's private taste over product design and supplier i's product characteristics, and q_i is called bidder i's quality in the remaining of this chapter. Assuming preference is quasilinear in price, then the auctioneer's utility from purchasing supplier i's product at price p_i is

$$U(q_i, p_i) = q_i - p_i \tag{1}$$

If the auctioneer does not disclose any information, then all suppliers have common belief that q_i is independently and identically distributed according to distribution $G(.)$ on $[\underline{q}, \bar{q}]$. Furthermore, assume $\underline{q} > g(0) + \bar{\eta}$ and there is no outside buying options so that it is always ex post efficient for the auctioneer to purchase the product from one of the potential suppliers.

t=3: The auctioneer sends a private signal $\hat{q}_i \in \{\{q_i\}_{i=1}^N, q_i, \emptyset\}$ to every bidder i according to the information policy chosen at $t = 1$.

t=4: After observing the signal provided by the auctioneer, each supplier i makes an investment $k_i \in \mathbb{R}^+$ simultaneously to reduce the cost of his product given common cost-reducing technology $g(.)$. k_i is the sunk cost of investment.

t=5: At the beginning of the auction stage, each supplier i receives a random cost shock η_i that is independently and identically distributed according to a commonly known uniform distribution $H(.)$ on $[\underline{\eta}, \bar{\eta}]$.

Following Dasgupta (1990) [6]'s production cost model, I assume the total production cost of supplier i is given by

$$c_i = c(k_i, \eta_i) = g(k_i) + \eta_i \tag{2}$$

in which $g' < 0$, $g'' > 0$, $\lim_{k \to 0} -g'(k) = \infty$, and $\lim_{k \to \infty} -g'(k) = 0$, so the cost reducing investment exhibits decreasing returns.

Each supplier i's "value" v_i as the total trading surplus that he can provide by selling the product to the auctioneer is therefore given by

$$v_i = v(q_i, k_i, \eta_i) = q_i - g(k_i) - \eta_i \tag{3}$$

t=6: Each supplier submits bid b_i that represents the minimum payment he is willing to accept to provide the product in a second score auction. The scoring rule used in the auction is

$$\sigma_i = q_i - b_i \tag{4}$$

The auctioneer announces scores of all the bidders at end of the auction. The highest-score bidder i wins the contract and receives a payment equals to the bid of the supplier j with the second highest score, adjusted by their quality difference: $p_i = b_j + q_i - q_j$.

3.2 Equilibrium of Second Score Auction

I will first show that truth-telling is still a dominant strategy for suppliers in the second score procurement auction when each supplier's value depends on the auctioneer's information q_i.

Proposition 1. *In the second score procurement auction with differentiated suppliers selling imperfect substitutes, it is still a dominant strategy for each supplier to bid his true production cost c_i.*

Proof. See Appendix. □

Since the auctionner privately knows the qualities of all bidders, by submitting a bid b_i, the value profile of all bidders $\{v_i\}_{i=1}^N$ will be revealed. Therefore, the second score auction can be written as a direct revelation mechanism in which the arguments of the allocation rule and the payment rule is the profile of bidders' values $\{v_i\}_{i=1}^N$.

3.3 Equilibrium of Investment Stage

In this section, I will characterize each supplier's optimal investment strategy. At the investment stage, the suppliers choose investment levels to maximize their expected payoffs in the auction, given that all suppliers report truthfully in the second score auction.

Define $\hat{F}_{-i}\left(.\middle|q_{-i}, k^*_{-i}\right)$ as the distribution of the highest value among bidder i's $(N-1)$ opponents' values at the beginning of the auction stage, before the random cost shocks η are realized. Then $\hat{F}_{-i}(.|q_{-i}, k^*_{-i})$ depends on opposing bidders' qualities q_{-i} and equilibrium investment strategies k^*_{-i}.

The incentive compatibility of the second score auction implies that the expected payoff of bidder i with value v_i is given by

$$\Pi_i(v_i) = \Pi_i(\underline{v}) + \int_{\underline{v}}^{v_i} \hat{F}_{-i}\big(\tau \big| q_{-i}, k^*_{-i}\big) d\tau \tag{5}$$

At the investment stage, each supplier chooses an investment level k_i^* that maximizes the expected payoff in the auction as a best response to opponents' investments k^*_{-i}, given the distribution of qualities and random cost shocks, and the information provided by the auctioneer $\hat{q}_i$.

Definition 1. *A profile of investments chosen at investment stage $\{k_i^*\}_{i=1}^N$ is an equilibrium under information provision scheme $\hat{q}_i$ if for all i,*

$$k_i^* \in \underset{k_i}{\operatorname{argmax}}\, E_{q,\eta}\Big[\int_{\underline{v}}^{v_i(q_i,k_i,\eta_i)} \hat{F}_{-i}\big(\tau \big| k^*_{-i}, q_{-i}\big) d\tau \Big| \hat{q}_i\Big] - k_i \tag{6}$$

Let $\hat{V}(v_1, v_2, \cdots, v_N)$ denote the second highest value given a profile of values $\{v_i\}_{i=1}^N$. Then $\hat{V}(v_1, v_2, \cdots, v_N)$ is the auctioneer's ex-post revenue given $\{v_i\}_{i=1}^N$.

The auctioneer's problem is to choose $\hat{q}_i \in \Big\{\{q_j\}_{j=1}^N, q_i, \emptyset\Big\}$ to induce a profile of values $(v_1, \cdots, v_N)$ that yields the highest ex ante expected revenue in the auction, given that suppliers will play equilibrium investment strategy in the investment stage given the information provided by the auctioneer.

Definition 2. *The auctioneer's problem of optimizing information provision is*

$$\begin{aligned} &\max_{\hat{q} \in \{\{q_j\}_{j=1}^N, q_i, \emptyset\}} E_\eta\Big[\hat{V}\big(v_1, v_2, \cdots, v_N\big)\Big] \\ s.t. \quad & v_i = q_i - g(k_i^*) - \eta_i \\ & k_i^* \in \underset{k_i}{\operatorname{argmax}}\, E_{q,\eta}\Big[\int_{\underline{v}}^{q_i - g(k_i) - \eta_i} \hat{F}_{-i}\big(\tau \big| k^*_{-i}, q_{-i}\big) d\tau \Big| \hat{q}_i\Big] - k_i \quad \forall i \end{aligned} \tag{7}$$

To study the impact of auctioneer's information provision of private valuations q_i, I will compare the suppliers' equilibrium investment strategies and expected revenues in the auction under concealment, private disclosure, and public disclosure. The main analysis will focus on the case where there are only $N = 2$ bidders. A discussion of the general case with $N \geq 2$ bidders will be provided in the end.

4 Equilibrium Investment Strategies with Two Sellers

In this section, I will analyze the suppliers' investment strategy when there are $N = 2$ bidders. Let i and j denote the identity of the 2 bidders. For each bidder i, the distribution of the opposing bidder j's value given bidder j's quality q_j and investment k_j^* at the beginning of the auction is given by

$$\begin{aligned} \hat{F}_{-i}\big(\tau \big| q_j, k_j^*\big) &= Prob\big(q_j - g(k_j^*) - \eta_j \leq \tau\big) \\ &= Prob\big(\eta_j \geq q_j - g(k_j^*) - \tau\big) \\ &= 1 - H\big(q_j - g(k_j^*) - \tau\big) \end{aligned} \tag{8}$$

Given the distribution of quality $G(q)$, let Q_1 and Q_2 denote the random variables that represent the highest order statistic and the second highest order statistic among bidders' qualities, respectively. Let (q_1, q_2) be realizations of (Q_1, Q_2). Then $q_1 = \max\{q_i, q_j\}$ and $q_2 = \min\{q_i, q_j\}$ for any realization of qualities $\{q_i, q_j\}$.

Define $\Delta(G)$ as the ex ante expected difference between q_1 and q_2 given distribution G:

$$\Delta(G) = E\big(q_1 - q_2 \big| G\big) \tag{9}$$

$\Delta(G)$ represents the expected dispersion of the auctioneer's valuation on the two bidders' products, which in turn measures how much the auctioneer cares about non-price characteristics relative to cost. Mathematically, $\Delta(G)$ represents the expected difference between the first order statistics and the second order statistics among 2 draws given distribution $G(.)$. Holding the expected quality constant, a greater $\Delta(G)$ implies that the expected difference between the higher quality and the lower quality is larger, i.e., the auctioneer is willing to pay more for contracting with the high-quality supplier instead of the low-quality supplier. When $\Delta(G) = 0$, $q_1 = q_2 = E(q|G)$, this model turns into the standard procurement auction model without product differentiation: the auctioneer's valuation for any supplier's product equals $E(q|G)$ and is common knowledge. There is no difference between the three information provision schemes when $\Delta(G) = 0$.

Since the three information disclosure policies yields the same expected revenue when $\Delta(G) = 0$, I will next explore how the expected revenue under the three disclosure policies change when holding the expected quality constant and increasing the dispersion of qualities $\Delta(G)$ in the following analysis.

4.1 Equilibrium under Concealment of Quality

Under concealment of qualities, each supplier chooses investment strategy knowing only the distribution of (q_i, q_j) and distribution of (η_i, η_j). Note that at the time of investment, suppliers are ex ante identical with symmetric distribution of q_i and η_i.

Given any level of opponent's investment k_j, each bidder i chooses investment k_i^* that solves

$$\max_{k_i} \int_{q_i} \int_{q_j} \int_{\underline{v}}^{q_i - g(k_i) - E\eta_i} \Big\{1 - H\big(q_j - g(k_j) - \tau\big)\Big\} d\tau dG(q_j) dG(q_i) - k_i \tag{10}$$

Take the first order condition will give supplier i's best response investment function $k_i^*(k_j)$ to the opponent's investment k_j. A subgame perfect equilibrium (k_i^C, k_j^C) is given by $k_i^C = k_i^*(k_j^C)$ and $k_j^C = k_j^*(k_i^C)$. By examining the first order condition and the second order condition of equation (10), the next proposition shows that the two bidders will chose identical investment k^C in equilibrium, in which k^C depends only on the cost reducing technology $g(.)$.

Proposition 2. *Under concealment of quality with $N = 2$, both suppliers will select an identical investment $k_i^C = k_j^C = k^C$ in a subgame perfect equilibrium. k^C does not depend on $G(.)$.*

Proof. See Appendix. □

Proposition 2 comes from the ex ante symmetry across bidders at the time when they make investment decisions. At the optimal level of investment, the marginal expected return from investment should equal the marginal cost of investment, given that the opponent also invests optimally. Given the ex-ante symmetry of the bidders, the expected return of investment in auction is always equivalent for two bidders, and the marginal cost of investment depends only on technology $g(.)$. Therefore, the equilibrium investment k^C is identical across bidders and is independent of the quality distribution $G(q)$.

4.2 Equilibrium under Private Disclosure of Quality

Under private disclosure of quality, suppose a symmetric perfect Bayesian equilibrium investment strategy $k^D : [\underline{q}, \bar{q}] \rightarrow \mathbb{R}^+$ exists. Each supplier's optimal investment strategy k_i^D solves

$$\max_{k_i} \int_{q_j} \int_{\underline{v}}^{q_i - g(k_i) - E\eta_i} \Big\{ 1 - H(q_j - g\big(k^D(q_j)\big) - \tau) \Big\} d\tau dG(q_j) \tag{11}$$

The equilibrium investment strategy of each bidder $k_i^D = k^D(q_i)$ is characterized by the first order condition of i's objective function given in equation (11).

The next proposition shows that privately disclosing quality q_i to each bidder will induce ex ante high quality suppliers to invest more aggressively compared to low quality suppliers. The symmetric equilibrium investment strategy $k^D(q_i)$ is increasing in q_i.

Proposition 3. *When there are only 2 bidders, under private disclosure of quality, the perfect Bayesian equilibrium investment $k^D(q_i)$ is increasing in q_i.*

Proof. See Appendix. □

Proposition 3 comes from the fact that the optimal investment decision of each bidder depends on the expected return of investment in the auction. Suppliers with higher quality products has higher expected probability of winning the auction than suppliers with lower quality products. The former has higher expected return for any given level of investment.

4.3 Equilibrium under Public Disclosure of Quality

Now suppose the auctioneer publicly announce the entire quality profile $\{q_i, q_j\}$ at the beginning of investment stage to all bidders. Under the public disclosure of quality, each bidder will hold different belief over the distribution of its opponent's value. Given $\{q_i, q_j\}$, and any level of opponent's invesment k_j, each bidder i will choose investment strategy k_i^* that solves

$$\max_{k_i} \int_{\underline{v}}^{q_i - g(k_i) - E\eta_i} \Big(1 - H\big(q_j - g(k_j) - \tau\big)\Big) d\tau \tag{12}$$

Under public disclosure of (q_i, q_j) to each bidder, the best response investment $k_i^*(k_j; q_i, q_j)$ to opponent's investment k_j is characterized by the first order condition of i's objective function given by equation (12) with k_j^A replaced by k_j. Let (k_i^A, k_j^A) denote the subgame perfect equilibrium investment profile under public disclosure (announcement) of qualities. For any quality profile (q_i, q_j), the subgame perfect equilibrium investment profile under public information disclosure (k_i^A, k_j^A) is defined as $k_i^A(q_i, q_j) = k_i^*(k_j^A; q_i, q_j)$ and $k_j^A(q_i, q_j) = k_j^*(k_i^A; q_i, q_j)$, in which $k_i^*(.; q_i, q_j)$ and $k_j^*(.; q_i, q_j)$ are each bidder's best response function.

The next proposition shows that given the same cost reducing technology $g(.)$, publicly disclosing all bidders' qualities will further induce the high quality supplier to invest more aggressively, and the low quality supplier to invest less aggressively. Each bidder's equilibrium investment $k_i^A(q_i, q_j)$ under announcement of entire quality profile is increasing in $(q_i - q_j)$.

Proposition 4. *When there are only 2 bidders, under public disclosure of qualities (q_i, q_j), each bidder's best response investment $k_i^*(k_j; q_i, q_j)$ is increasing in $(q_i - q_j)$ and decreasing in k_j. The subgame perfect equilibrium investment $k_i^A(q_i, q_j)$ is increasing in $(q_i - q_j)$.*

Proof. See Appendix. □

Proposition 4 comes from the fact that the higher quality bidder has higher expected return from investment, as the expected probability of winning the auction is higher. When the higher quality bidder knows exactly his ex ante advantage before the auction starts, his investment incentive will be stronger, while the lower quality bidder will be discouraged from investing given this information. This is because the pre-auction investments are strategic substitutes between bidders, and knowing that opponent has a low quality for certain will make the high quality bidder believe that the investment of the opponent is also low, which further increases the expected return from investment.

4.4 Revenue Comparison

At the beginning of auction, the expected value of bidder i with quality q_i and investment k_i before the realization of random cost component η_i is given by

$$\begin{aligned} V(k_i, q_i) &= \int_{\underline{\eta}}^{\bar{\eta}} \big(q_i - g(k_i) - \eta_i\big) dH(\eta_i) \\ &= q_i - g(k_i) - E\eta_i \end{aligned} \tag{13}$$

Define $V(k_1^L, q_1)$ and $V(k_2^L, q_2)$ as the equilibrium expected value of the high quality supplier and the low quality supplier under information policy $L \in \{C, D, A\}$, in which C represents concealment, D represents private disclosure and A represents public disclosure (announcement), at the beginning of auction, given their equilibrium investments k_1^L, k_2^L under realizations $Q_1 = q_1, Q_2 = q_2$:

$$\begin{aligned} V(k_1^L, q_1) &= q_1 - g(k_1^L) - E\eta \\ V(k_2^L, q_2) &= q_2 - g(k_2^L) - E\eta \end{aligned} \tag{14}$$

Under concealment of qualities, $k_1^C = k_2^C = k^C$. Under private disclosure of qualities, $k_1^D = k^D(q_1)$ and $k_2^D = k^D(q_2)$. Under public disclosure of qualities, $k_1^A = k_1^A(q_1, q_2)$ and $k_2^A = k_2^A(q_2, q_1)$.

The ex ante expected winner's payoff in the auction under policy $L \in \{C, D, A\}$ given distribution G is given by

$$E\Pi^L(G) = E[V(k_1^L, q_1) - V(k_2^L, q_2)|G] \tag{15}$$

The ex ante expected revenue to the auctioneer under policy $L \in \{C, D, A\}$ given distribution G is given by

$$ER^L(G) = E[V(k_2^L, q_2)|G] \tag{16}$$

As mentioned at the beginning of this chapter, when the auctioneer does not care about non-price characteristics and $\Delta(G) = 0$, the three information disclosure policy gives the same expected revenue:$ER^C(G) = ER^D(G) = ER^A(G)$. I will next analyze how the expected revenues change under the three different information provision policies as $\Delta(G)$ increases from 0 in order to compare the revenues of the three information provision policies when $\Delta(G) > 0$.

The next proposition shows that the expected revenues $ER^C(G)$, $ER^D(G)$ and $ER^A(G)$ are decreasing in $\Delta(G)$ under all three information provision schemes, when holding the expected quality constant. It can be shown that the negative impact of increasing $\Delta(G)$ on $ER^C(G)$ is weaker than that on $ER^D(G)$ and $ER^A(G)$, at any level of $\Delta(G) > 0$. This implies that when there are only 2 bidders, the ex ante expected revenue to the auctioneer is always highest under concealment of quality among the three information schemes.

Proposition 5. *When there are only 2 bidders, the expected revenue to the auctioneer $ER^L(G)$ is decreasing in $\Delta(G)$ for all $L \in \{C, D, A\}$. Moreover,*

$$\begin{aligned} \frac{dER^A(G)}{d\Delta(G)} < \frac{dER^C(G)}{d\Delta(G)} < 0 \\ \frac{dER^D(G)}{d\Delta(G)} < \frac{dER^C(G)}{d\Delta(G)} < 0 \end{aligned} \tag{17}$$

When $\Delta(G) = 0$, $ER^C(G) = ER^D(G) = ER^A(G)$

When $\Delta(G) > 0$, $ER^C(G) > ER^D(G)$ and $ER^C(G) > ER^A(G)$. Both $ER^C(G) - ER^D(G)$ and $ER^C(G) - ER^A(G)$ are increasing in $\Delta(G)$.

Proof. See Appendix. □

Proposition 5 implies that when there are only 2 bidders, it is always optimal for the auctioneer to conceal their qualities. When the auctioneer discloses her private values to the bidders, the lower quality bidder will be discouraged from making investments, which leads to lower expected value of the lower quality bidder and lower expected revenue in the auction.

The result of Proposition 5 comes from the fact that $\Delta(G)$ represents the dispersion of quality distribution G. Holding the expected quality constant and increasing $\Delta(G)$ will generate a mean preserving spread of the original distribution, under which it is more likely to observe a high value of q_1 and a low value of q_2. This is the only source that drives the fact that $E\Pi^C(G)$ being increasing in $\Delta(G)$ and $ER^C(G)$ being decreasing in $\Delta(G)$ under concealment of quality, as the equilibrium investment k^C is independent of G. This source also present under private disclosure of quality and public disclosure of quality. However, under private disclosure and public disclosure of quality, increasing $\Delta(G)$ will not only decrease the expected value of q_2, but also decrease the expected investment of the lower quality bidder, as the low quality bidder will be discouraged from investment by receiving a low quality signal. Therefore, the impact of $\Delta(G)$ on expected revenue is stronger when the auctioneer discloses her values than that when the auctioneer conceals her values. Moreover, the difference between expected revenues under any two information schemes is increasing in $\Delta(G)$, as the bidders' investment incentives will be affected by the information provided by the auctioneer more significantly when auctioneer cares more about non-price characteristics.

5 Conclusions

This paper studies the information provision problem in a procurement auction where the auctioneer has private subjective valuations over the suppliers' products, and suppliers have opportunity to invest in cost reduction prior to entering the auction. In this paper, I analyze the equilibrium investment strategies of suppliers under concealment of auctioneer's private valuations, private disclosure of auctioneer's valuation, and public disclosure of auctioneer's valuations, and provide a revenue comparison among these three information provision schemes under the presence of 2 bidders. The main conclusions are summarized as below:

First, disclosing the auctioneer's private valuation over each supplier's quality will induce high quality suppliers to invest more aggressively and discourage low quality suppliers from making investments. This result comes from the fact that each bidder's expected return from investment is increasing in his quality. Therefore, providing more information will induce a more dispersed distribution of values in the auction through this differentiation effect at the investment stage. When there are only two bidders, providing more information will discourage the lower quality bidder from investment and reduce the expected revenue. This leads to the result that concealment gives the highest expected revenue among the three information provision schemes considered in this paper.

Second, when one information scheme dominates the other information scheme under given distribution G, the benefit of the better scheme over the worse scheme increases in the dispersion of quality $\Delta(G)$. This result comes from the fact that

$\Delta(G)$ measures how much the auctioneer cares about qualities relative to costs. When the auctioneer cares more about qualities, the impact of quality differentiation on bidders' investment incentives is stronger, and providing information on this quality differentiation has greater impact on the equilibrium distribution of values.

I will next provide a brief discussion on the more general case when there are $N \geq 2$ bidders. Define $\Delta(G, N) = E[q_1 - q_N|G, N]$. Then $\Delta(G, N)$ measures the dispersion of qualities among N bidders given distribution $G(.)$. It is natural to conjecture that given a fixed number of bidders N and distribution G, concealment is optimal only if N is small enough s.t. the expected value of second order statistics is decreasing in the dispersion of qualities. When N is large enough s.t. $E(q_2|G, N)$ is increasing in $\Delta(G, N)$, then the rank order of expected revenues under three information provision policies will be reversed, and public disclosure will provide the highest expected revenue. When N approaches infinity, it is always optimal to publicly disclose all qualities. This result is consistent with Ganuza (2004) [7]'s finding that the optimal level of information provision is increasing in the number of bidders. When there are only 2 bidders, competition in auction is weak, and disclosing the auctioneer's private information will give more informational rent to the winner. In contrast, when the number of bidders is large, disclosing more information will promote competition among the high quality bidders and will increase the expected revenue. When the number of bidders approaches infinity so that the model approaches a perfectly competitive market where each seller captures zero informational rent, it is optimal for the auctioneer to disclose all information.

The findings in this paper suggest a few directions for future research. First, this paper assumes that participation in the auction is costless and the number of bidders in the auction N is exogenous. Since the provision of the auctioneer's information also changes the ex ante expected payoff to the winner, it would be interesting to allow endogenous entry of bidders. If the quality information is disclosed before bidders make entry decisions, then low quality bidders will not enter the auction, which reduces the degree of competition and lowers the auctioneer's expected revenue. When disclosing more information is optimal under exogenous entry, the positive impact of information disclosure on expected revenue through inducing higher quality bidders investing more will be offset by the negative impact through preventing low quality bidders from participating. On the other hand, when there are very few bidders so that concealing information is optimal given this fixed number of bidders, disclosing information will yield higher expected payoff to the winner and therefore induce more bidders to enter, so the optimal information disclosure scheme again becomes ambiguous. The next step of this research may introduce entry cost to the model and study how the revenue ranking of three information provision schemes change when number of bidders is also endogenously determined by the information provision scheme.

Second, this paper assumes that providing information to bidders is costless to the auctioneer, which is not a practical assumption, as communication between the auctioneer and bidders usually comes at a cost. When providing information is costly, the benefit of information disclosure to the auctioneer may be outweighed by the cost of communication. When the cost of information provision is independent of number of bidders, it would be optimal to disclose quality when N is large enough since the benefit of information provision increases in N. However, when the cost of information provision also increases in N, the optimal level of information provision becomes ambiguous, and the next step of this study may include providing a characterization of the optimal level of information provision when providing information is costly.

Appendix

Proof of Proposition 1:

Proof. Let b_i denote the bid submitted by bidder i. Since q_i is known to the auctioneer, define the adjusted bid of i as $\hat{v}_i = q_i - b_i$. The true value of bidder i is given by $v_i = q_i - c_i$. Reporting $b_i > c_i$ will lead to $\hat{v}_i < v_i$ and losing the auction when the supplier could have profitably won the auction with $\hat{v}_i = v_i$. Reporting $b_i < c_i$ will lead to $\hat{v}_i > v_i$ and winning the auction with negative payoff when $v_i - \hat{v}_j < 0$. Therefore, as in standard second-price auctions, it is a dominant strategy for each supplier to report true value v_i by submitting bid equals to marginal cost c_i truthfully. □

Proof of Proposition 2:

Proof. Under the concealment of quality q_i, each supplier's objective function at the investment stage is

$$\max_{k_i} \int_{q_i} \int_{q_j} \int_{\underline{v}}^{q_i - g(k_i) - E\eta_i} \Big\{ 1 - H(q_j - g(k_j^*) - \tau) \Big\} d\tau dG_j(q_j) dG_i(q_i) - k_i \tag{18}$$

The first order condition is given by

$$-g'(k_i^*) \times \underbrace{\left\{ \int_{q_i} \int_{q_j} \Big\{ 1 - H(q_j - g(k_j^*) - q_i + g(k_i^*) + E\eta_i) \Big\} dG(q_j) dG(q_i) \right\}}_{\text{expected probability of winning}} - 1 = 0 \tag{19}$$

Given the symmetry of the two bidders, the first order condition for the bidders is symmetric, which means we must have $k_i^* = k_j^*$ in equilibrium.

Since bidders are ex ante identical, in any symmetric equilibrium, the ex-ante expected probability of winning the auction is always $\frac{1}{2}$, i.e.,

$$\int_{q_i} \int_{q_j} \Big\{ 1 - H(q_j - g(k_j^*) - q_i + g(k_i^*) + E\eta_i) \Big\} dG(q_j) dG(q_i) = \frac{1}{2} \tag{20}$$

The first order condition can be therefore written as

$$\begin{aligned} & -g'(k_i^*) \times \frac{1}{2} - 1 = 0 \\ & -g'(k_i^*) = 2 \end{aligned} \tag{21}$$

The symmetric equilibrium investment under concealment of quality $k^C = k_i^* = k_j^*$ is therefore independent of the distribution $G(.)$ and $H(.)$. For any given cost reducing technology $g(.)$, the equilibrium investment k^C under quality concealment is identical across bidders and identical under any distribution of quality G. □

Proof of Proposition 3:

Proof. When the auctioneer privately discloses q_i, each supplier i's objective function is

$$\max_{k_i} \int_{q_j} \int_{\underline{v}}^{q_i - g(k_i) - E\eta_i} \Big\{1 - H\big(q_j - g(k^D(q_j)) - \tau\big)\Big\} d\tau dG(q_j) - k_i \tag{22}$$

The first order condition of each supplier's objective function is

$$-g'(k_i^D) \times \int_{q_j} \Big\{1 - H\big(q_j - g\big(k^D(q_j)\big) - q_i + g(k_i^D) + E\eta_i\big)\Big\} dG(q_j) - 1 = 0 \tag{23}$$

Suppose SOC < 0 s.t. an equilibrium exists. $k_i^D = k^D(q_i)$ characterized by FOC is the equilibrium investment strategy of supplier i with quality q_i. Take total differentiation of FOC with respect to k_i^D and q_i:

$$\frac{dk_i^D}{dq_i} = -\frac{\int_{q_j} H'\big(q_j - g\big(k^D(q_j)\big) - q_i + g(k_i^D) + E\eta_i\big)(-g'(k_i^D)) dG(q_j)}{SOC} > 0 \tag{24}$$

since $H'(.) > 0$, $-g'(.) > 0$, and the denominator < 0 by second order condition. Therefore, the equilibrium investment k_i^D is increasing in each supplier's quality q_i when the auctioneer discloses q_i at the investment stage.

□

Proof of Proposition 4:

Proof. Under public disclosure of qualities, the objective function for bidder i given information (q_i, q_j) and the opponent's investment k_j is

$$\max_{k_i} \int_{\underline{v}}^{q_i - g(k_i) - E\eta_i} \Big\{1 - H\big(q_j - g(k_j) - \tau\big)\Big\} d\tau - k_i \tag{25}$$

Each bidder i's best response investment $k_i^*(k_j; q_i, q_j)$ to any level of opponent's investment k_j is characterized by

$$-g'(k_i^*) \times \Big\{1 - H\big(q_j - g(k_j) - q_i + g(k_i^*) + E\eta_i\big)\Big\} - 1 = 0 \tag{26}$$

Take total differentiation of the best response function with respect to k_i^* and $\big(q_i - q_j\big)$:

$$\frac{\partial k_i^*}{\partial (q_i - q_j)} = -\frac{H'\big(-(q_i - q_j) + g(k_i^*) - g(k_j) + E\eta_i\big)\big(-g'(k_i^*)\big)}{SOC} > 0 \tag{27}$$

since $H'(.) > 0$, $-g'(k_i^*) > 0$, and $SOC < 0$. Therefore, the best response investment of i to any investment level of j will shift to the right when quality difference $(q_i - q_j)$ increases.

Take total differentiation of FOC with respect to k_i^* and k_j:

$$\frac{\partial k_i^*}{\partial k_j} = -\frac{H'\big(q_j - g(k_j) - q_i + g(k_i^*) + E\eta_i\big)g'(k_j)\big(-g'(k_i^*)\big)}{SOC} < 0 \tag{28}$$

since $H'(.) > 0$, $-g'(k_i^*) > 0$, $g'(.) < 0$, and SOC < 0. So the best response investment of i is decreasing in the opponent's investment k_j under any announced quality (q_i, q_j).

The intersection of $k_i^*(k_j; q_i, q_j)$ and $k_j^*(k_i; q_i, q_j)$ gives the equilibrium investments (k_i^A, k_j^A). For each bidder i, assuming the opponent is playing the equilibrium k_j^A, then k_i^A is characterized by the first order condition given by

$$-g'(k_i^A) \times \Big\{1 - H\big(q_j - g(k_j^A) - q_i + g(k_i^A) + E\eta_i\big)\Big\} - 1 = 0 \tag{29}$$

Suppose SOC < 0 s.t. an equilibrium exists. Take total differentiation of FOC with respect to k_i^A and $\big(q_i - q_j\big)$:

$$\frac{dk_i^A}{d(q_i - q_j)} = -\frac{H'\big(-(q_i - q_j) + g(k_i^A) - g(k_j^A) + E\eta_i\big)\big(-g'(k_i^A)\big)}{SOC} > 0 \tag{30}$$

since $H'(.) > 0$, $-g'(k_i^A) > 0$, and $SOC < 0$. Therefore, the equilibrium investment of supplier i is increasing in the announced quality difference $(q_i - q_j)$. □

Proof of Proposition 5:

Proof. The expected revenue under concealment of quality is given by

$$ER^C(G) = E\Big[V(k^C, q_2)\Big|G\Big] = E\Big[q_2 - g(k^C) - E\eta\Big|G\Big] \tag{31}$$

in which k^C is independent of G and q. The total effect of $\Delta(G)$ on $ER^C(G)$ is given by

$$\begin{aligned}\frac{dER^C(G)}{d\Delta(G)} &= \frac{dE\Big[q_2 - g(k^C) - E\eta\Big|G\Big]}{d\Delta(G)} \\ &= \frac{dE(q_2|G)}{d\Delta(G)} < 0\end{aligned} \tag{32}$$

The expected revenue to the auctioneer under private disclosure of quality is given by

$$ER^D(G) = E\Big[V(k_2^D, q_2)\Big|G\Big] = E\Big[q_2 - g(k^D(q_2)) - E\eta\Big|G\Big] \tag{33}$$

Holding the expected quality constant and increasing the dispersion $\Delta(G)$ will decrease the expected value of the low quality and decrease the expected investment of the low quality supplier. The total impact of $\Delta(G)$ on $ER^D(G)$ is

$$\begin{aligned}\frac{dER^D(G)}{d\Delta(G)} &= \frac{dE\Big[q_2 - g(k^D(q_2)) - E\eta\Big|G\Big]}{d\Delta(G)} \\ &= \frac{dE(q_2|G)}{d\Delta(G)} - g'(k_2^D)k^{D'}(q_2)\frac{dE(q_2|G)}{d\Delta(G)} \\ &= \frac{dE(q_2|G)}{d\Delta(G)} \times \big\{1 - g'(k_2^D)k^{D'}(q_2)\big\} < 0\end{aligned} \tag{34}$$

Since $1 - g'(k_2^D)k_2^{D'}(q_2) > 1$,

$$\frac{dER^D(G)}{d\Delta(G)} < \frac{dER^C(G)}{d\Delta(G)} \tag{35}$$

i.e., the negative impact of increased dispersion in G on ER^D is greater than on ER^C. Subtracting $ER^D(G)$ from $ER^C(G)$ gives

$$\frac{d(ER^C(G) - ER^D(G))}{d\Delta(G)} = g'(k_2^D)k^{D'}(q_2)\frac{dE(q_2|G)}{d\Delta(G)} > 0 \tag{36}$$

which also implies that the difference in expected qualities under concealment and under private disclosure is increasing in $\Delta(G)$.

The expected revenue to the auctioneer under public disclosure of quality is

$$ER^A(G) = E\Big[V(k_2^A, q_2)\Big|G\Big] = E\Big[q_2 - g(k_2^A(q_2, q_1)) - E\eta\Big|G\Big] \tag{37}$$

Holding the expected quality constant and increasing the dispersion $\Delta(G)$ will increase the expected difference $(q_1 - q_2)$ and decrease the expected investment of the low quality supplier. The total impact of $\Delta(G)$ on $ER^A(G)$ is

$$\begin{aligned}\frac{dER^A(G)}{d\Delta(G)} &= \frac{dE\Big[q_2 - g(k_2^A(q_2, q_1)) - E\eta\Big|G\Big]}{d\Delta(G)} \\ &= \frac{dE(q_2|G)}{d\Delta(G)} - g'(k_2^A)\frac{dk_2^A(q_2, q_1)}{d(q_2 - q_1)}\frac{dE(q_2 - q_1|G)}{d\Delta(G)} \\ &= \frac{dE(q_2|G)}{d\Delta(G)} - g'(k_2^A)\frac{dk_2^A(q_2, q_1)}{d(q_2 - q_1)}(-1) < 0\end{aligned} \tag{38}$$

Subtracting $ER^A(G)$ from $ER^C(G)$ gives

$$\frac{d(ER^C(G) - ER^A(G))}{d\Delta(G)} = g'(k_2^A)\frac{dk_2^A(q_2, q_1)}{d(q_2 - q_1)}(-1) > 0 \tag{39}$$

Therefore, $\frac{dER^A(G)}{d\Delta(G)} < \frac{dER^C(G)}{d\Delta(G)}$, and the difference in expected revenues is increasing in $\Delta(G)$.

Since $ER^C(G) = ER^D(G) = ER^A(G)$ when $\Delta(G) = 0$ and

$$\begin{aligned}\frac{dER^A(G)}{d\Delta(G)} &< \frac{dER^C(G)}{d\Delta(G)} < 0 \\ \frac{dER^D(G)}{d\Delta(G)} &< \frac{dER^C(G)}{d\Delta(G)} < 0\end{aligned} \tag{40}$$

We have

$$ER^C(G) > ER^D(G), \quad \text{and} \quad ER^C(G) > ER^A(G) \tag{41}$$

for any distribution $G(.)$ that satisfies $\Delta(G) > 0$ when there are 2 bidders, and the difference in expected revenues is increasing in $\Delta(G)$. □

References

[1] Asker, J. and Cantillon, E. (2008): "Properties of Scoring Auctions," *RAND Journal of Economics*, Vol. 39, No. 1, Spring 2008, pp. 69-85

[2] Arozamena, L. and E. Cantillon (2004): "Investment Incentives in Procurement Auctions," *Review of Economic Studies*, 71(1), 1-18

[3] Bag, P. (1997): "Optimal Auction Design and R & D", *European Economic Review*, 41, 1655–1674

[4] Coleff, J. and Garcia, D. (2014): "Information provisions in procurement auctions," Working paper

[5] Colucci, D., Doni, N. and Valori, V. (2015): "Information policies in procurement auctions with heterogeneous suppliers," *Journal of Economics*, 2015, Volume 114, Number 3, Page 211-238

[6] Dasgupta, S. (1990): "Competition for Procurement Contracts and Investment," *International Economic Review*, November 1990, Volume 31 , No. 4, pp. 841-865

[7] Ganuza, J. (2004): "Ignorance Promotes Competition: An Auction Model with Endogenous private Valuations," *The RAND Journal of Economics*, 35, 583-598

[8] Hatfield, J. W., Kojima, F. and Kominers, S.D. (2015): "Strategy-Proofness, Investment Efficiency, and Marginal Returns: An Equivalence," Beckerman Institute for Research in Economics, Working Paper.

[9] Li, Y., Lovejoy, W.S. and Gupta, S. (2006): "Pre-auction Investments by Type-conscious agents," *SSRN Electronic Journal*, August 2006.

[10] Lichtenberg, F.R. (1986): "Private Investment in R & D to Signal Ability to Perform Government Contracts," NBER Working Paper Series No. 1974

[11] Milgrom, P.R. and Weber, R.J. (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50(5), 1089-1122

[12] Myerson, R. (1981): "Optimal Auction Design," *Mathematics of Operations Research*, 6, 58-73

[13] Piccione, M. and Tan, G. (1996): "Cost-Reducing Investment, Optimal Procurement and Implementation by Auctions," *International Economic Review*, 37(3), 663-685

[14] Thomas, C.J. and Wilson, B.J.(2012): "Horizontal Product Differentiation in Auctions and Multilateral Negotiations," *Economica*, Vol. 81, Issue 324, pp. 768-787, 2014

[15] Tomoeda, K. (2015): "Implementation of Efficient Investments in Mechanism Design," Working Paper.